UPPSC-AE 2025

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Electrical Engineering

Analog Communication and Microwave

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Analog Communication and Microwave

Contents

❍❍❍❍

CHAPTER 2

Amplitude Modulation

2.1 Introduction

In amplitude modulation, the amplitude of a **carrier** signal is varied in accordance with the instantaneous value of **modulating voltage,** whose frequency is invariably lower than that of the carrier. In practice, the carrier may be high-frequency (HF) while the modulating signal is audio.

After studying the theory of amplitude modulation techniques, one will be able to know that an AM wave is made of a number of frequency components having a specific relation to one another. Based on this observation, AM can be further classified as double sideband full carrier (DSBFC), double sideband suppressed carrier (DSBSC), single sideband (SSB) and vestigial sideband (VSB) modulation techniques. This is based on how many components of the basic amplitude modulated signal are chosen for transmission. This is followed by a description of different methods for the generation of AM, DSBSC, SSB and VSB signals.

2.2 Amplitude Modulation (AM)

Amplitude modulation is defined as a process in which the amplitude of the carrier wave $c(t)$ is varied linearly with the message signal $m(t)$ keeping other parameters constant.

2.2.1 Equation for AM

Let, $c(t) = A_C \cos \omega_c t$ .... is a carrier wave having; A_C = carrier amplitude; w_c = carrier frequency and m(t) = Base-band modulating signal band limited to maximum frequency "f_m"

Here, $$\omega_c = 2\pi f_c\ ;\ f_e >> f_m$$

Now, according to amplitude modulation the maximum amplitude 'A' of the carrier will have to be made proportional to the instantaneous amplitude of modulating signal m(t).

The standard equation for AM wave is described as

$$X_{AM}(t) = m(t) \cos\omega_C t + A_c \cos\omega_C t$$

$$\Rightarrow \quad \boxed{X_{AM}(t) = [A_C + m(t)]\cos\omega_c t}$$

$$= A_C [1 + K_a\, m(t)] \cos 2\pi f_c\, t$$

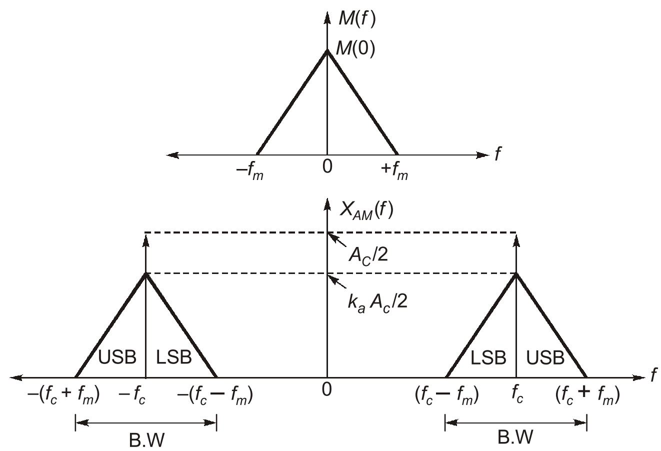
where, K_a = constant called amplitude sensitivity of modulator

For our sake of convenience let us take,

$$X_{AM}(t) = A_C \cos 2\pi f_c\, t + m(t) \cos 2\pi f_c\, t$$

$$X_{AM}(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{K_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

2.2.2 Spectrum of AM Signal



$$B.W = (\omega_c + \omega_m) - (\omega_c - \omega_m)$$

$$\boxed{B.W = 2\omega_m} \text{ rad/sec}$$

$$\boxed{B.W \approx 2f_m} \text{ Hz or kHz}$$

2.2.3 Single Tone Amplitude Modulation (Sinusoidal AM)

$$c(t) = A_C \cos \omega_c t \text{ ....carrier signal}$$

$$m(t) = A_m \cos \omega_m t \text{ .... modulating signal}$$

then after modulation, we get

$\therefore$
$$\boxed{X_{AM}(t) = [A_C + A_m \cos\omega_m t] \cos\omega_c t}$$

$$X_{AM}(t) = A_C\left[1 + \frac{A_m}{A_c} \cos \omega_m t\right] \cos\omega_c t$$

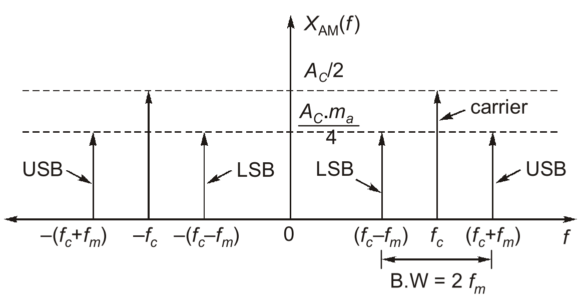
$$\boxed{X_{AM}(t) = A_C[1 + m_a \cos\omega_m t] \cos\omega_c t}$$

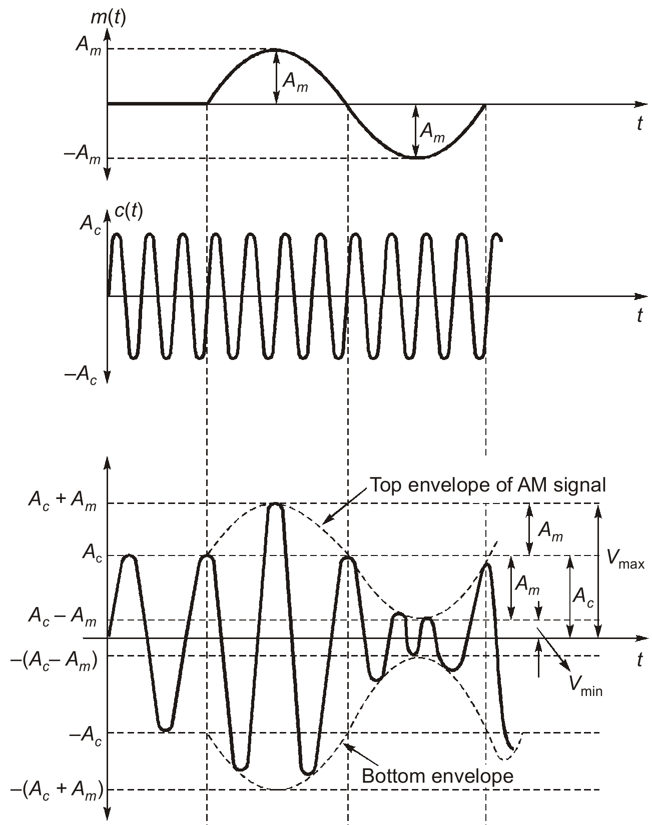
where, $m_a = \dfrac{A_m}{A_C}$ = **Modulation Index or Depth of modulation.**

$X_{AM}(t)$ can also be written as,

$$\boxed{X_{AM}(t) = \underbrace{A_c \cos\omega_c t}_{\text{Full carrier}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}}}$$

2.2.4 Spectrum of sinusoidal AM signal





$$2A_m = V_{max} - V_{min}$$

$$\Rightarrow \quad \boxed{A_m = \frac{V_{max} - V_{min}}{2}}$$

$$A_c = V_{max} - A_m$$

$$A_c = V_{max} - \frac{V_{max} - V_{min}}{2}$$

$$\Rightarrow \quad \boxed{A_c = \frac{V_{max} + V_{min}}{2}}$$

Finally we get,

$$\boxed{m_a = \frac{A_m}{A_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}} \rightarrow \text{modulation index}$$

- % modulation = $m_a \times 100$
- Modulation index gives the depth to which the carrier signal is modulated.
- For m(t) to be preserved in the envelope of AM signal, $\boxed{m_a \le 1}$

  i.e. $\boxed{A_m \le A_C}$

  So, range of m_a is, $\boxed{0 \le m_a \le 1}$

Example - 2.1 In an AM wave, V_{max} = 10 V and V_{min} = 5 V. The percentage of modulation is

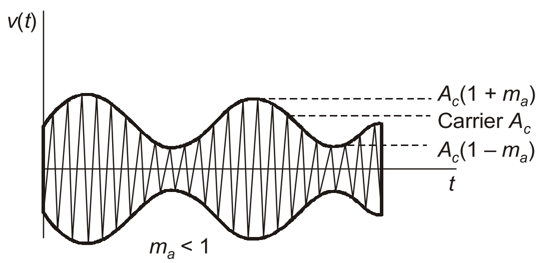
(a) 20.00 (b) 33.33
(c) 50.00 (d) 75.00 [UPPSC]

Answer: (b)

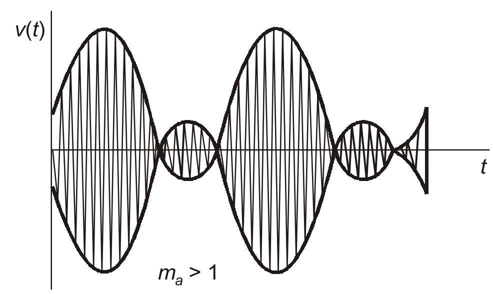
$$\text{Percentage of modulation } (\mu) = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{5}{15} = \frac{1}{3} = 33.33\%$$

2.2.5 Overmodulation

When $\boxed{m_a > 1}$ i.e. $\boxed{A_m > A_C}$ Over modulation takes place and the signal gets distorted. Because, the (–ve) part of wave form gets cut from the waveform leaving behind a "square wave type" of signal, which generates infinite number of harmonics. This type of distortion is known as "Non-linear distortion" or "Envelope distortion".



Undermodulated AM wave



Over modulated AM wave

2.2.6 Power (normalized) Relations in AM Wave

$$\boxed{P_{Total} = P_T = P_{carrier} + P_{USB} + P_{LSB}}$$

We know, $$\text{power} = \frac{V_{rms}^2}{R} = \left(\frac{V_m}{\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{V_m^2}{2} \cdot \frac{1}{R}$$

Here, we consider R = 1 for normalized power

$\therefore$ $$\text{Power} = \frac{A_C^2}{2} \cdot \frac{1}{R} \approx \frac{A_C^2}{2}$$

Now, we can write,

$$P_T = \frac{A_C^2}{2} + \frac{\left(\frac{m_a A_C}{2}\right)^2}{2} + \frac{\left(\frac{m_a A_C}{2}\right)^2}{2}$$

$\Rightarrow$ $$\boxed{P_T = \frac{A_C^2}{2}\left[1 + \frac{m_a^2}{2}\right] = P_C\left(1 + \frac{m_a^2}{2}\right)}$$

But in general,

$$\boxed{P_T = \frac{A_C^2}{2} \cdot \frac{1}{R}\left(1 + \frac{m_a^2}{2}\right)}$$

$$\Rightarrow \qquad \boxed{P_T = P_C + P_{SB} = P_C + \frac{P_C\, m_a^2}{2}}$$

$$P_T = P_C\left[1+\frac{m_a^2}{2}\right]$$

$\Rightarrow$ Total side band power = Total useful power

i.e. $$\boxed{P_{SB} = \frac{P_C\, m_a^2}{2}}$$

Because, our information content is in the only LSB or USB not in carrier.

2.2.7 Transmission efficiency (η)

$$\eta = \frac{\text{total useful power}}{\text{total transmitted power}} = \frac{P_{SB}}{P_T}$$

$$\therefore \qquad \boxed{\eta = \frac{m_a^2}{2+m_a^2}\times 100\%}$$

Remember

Current Relations in AM Signal:

Let, $P_T = I_T^2 \cdot R$ and $P_C = I_C^2 \cdot R$

$$\therefore \qquad \boxed{\frac{I_T}{I_C} = \sqrt{\left(1+\frac{m_a^2}{2}\right)}}$$

2.2.8 Modulation by Several Sine Waves (Multiple-Tone Modulation)

Let: $c(t) = A_C \cos\omega_c t$; $m_1(t) = A_{m_1}\cos\omega_1 t$; $m_2(t) = A_{m_2}\cos\omega_2 t$

$\therefore$ Total modulating signal $= m(t) = m_1(t) + m_2(t)$

So, $$\boxed{X_{AM}(t) = A_C\left[1+m_1\cos\omega_1 t + m_2\cos\omega_2 t\right]\cos\omega_c t}$$

where, $m_1 = \dfrac{A_{m_1}}{A_C}$ and $m_2 = \dfrac{A_{m_2}}{A_c}$

Power relations:

$$\boxed{P_T = P_C\left[1+\frac{m_1^2}{2}+\frac{m_2^2}{2}+\ldots\ldots\ldots\right]} \Rightarrow \boxed{P_T = P_C\left[1+\frac{m_T^2}{2}\right]}$$

where, $m^2{}_T = m^2{}_1 + m^2{}_2 + m^2{}_3 + \ldots\ldots$

where, m_T = total or net modulation index

$$\therefore \qquad \boxed{m_T = \sqrt{m_1^2 + m_2^2 + \ldots\ldots + m_n^2}}$$

Example - 2.2 A carrier is modulated by two modulating waves *A* and *B* having modulation index of 0.6 and 0.8 respectively. The overall modulation index is

(a) 1.0 (b) 0.7
(c) 0.2 (d) 1.4 [UPPSC]

Solution: (a)

Given: $\mu_1 = 0.6$ and $\mu_2 = 0.8$

$$\text{Overall modulation index} = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.36 + 0.64} = 1$$

Example - 2.3 A 10 kW carrier is sinusoidally modulated by two carriers corresponding to a modulation index of 30% and 40% respectively. The total radiated power is

(a) 11.25 kW (b) 12.5 kW
(c) 15 kW (d) 17 kW

Solution: (a)

Given that: $P_c = 10$ kW ; $m_1 = 30\% = 0.3$; $m_2 = 40\% = 0.4$

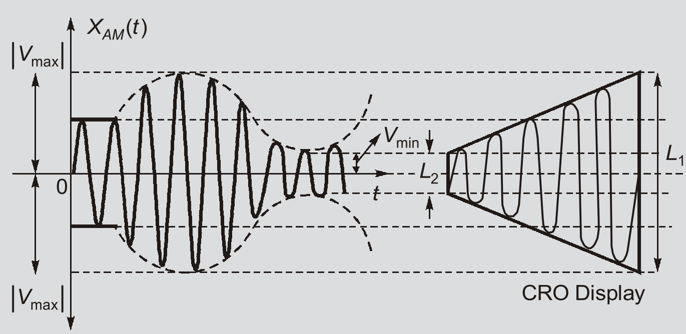
$\therefore$ $$\text{Total modulation index} = m_T = \sqrt{m_1^2 + m_2^2} = 0.5$$

Now, $$P_T = \text{total radiated power} = P_c\left(1 + \frac{m_T^2}{2}\right) = 11.25 \text{ kW}$$

NOTE

Trapezoidal display of AM Signal:

- modulated wave $\xrightarrow{\text{applied}}$ vertical deflection circuit of CRO.
- modulating wave $\xrightarrow{\text{applied}}$ horizontal deflection circuit of CRO.



Here, $L_1 = 2\,V_{max}$ and $L_2 = 2\,V_{min}$

So, $$\boxed{m_a = \text{modulation index} = \frac{L_1 - L_2}{L_1 + L_2}}$$

Example - 2.4 An AM voltage signal s(t), with a carrier frequency of 1.15 GHz has a complex envelope $g(t) = A_C\,[1 + m(t)]$, $A_C = 500$ V, and the modulation is a 1 kHz sinusoidal test tone described by $m(t) = 0.8 \sin(2\pi \times 10^3 t)$, appears across a 50 Ω resistive load. What is the actual power dissipated in the load?

(a) 165 kW (b) 82.5 kW
(c) 3.3 kW (d) 6.6 kW

Solution: (c)

Given that: $A_c = 500$ V ; $f_m = 10^3$ Hz = 1 kHz

Since,
$$g(t) = A_c[1 + m(t)]$$
$$g(t) = 500[1 + 0.8\sin(2\pi \times 10^3 t)$$

$\therefore$
$$m_a = 0.8$$
$$R_L = 50\ \Omega$$

Total actual power dissipated in the load,

$$P_T = \frac{A_c^2}{2}\cdot\left[1+\frac{m_a^2}{2}\right]\cdot\frac{1}{R} = \frac{500^2}{100}\left[1+\frac{0.8^2}{2}\right] = 3300 \text{ watt} = 3.3 \text{ kW}$$

2.3 Generation of AM Waves

- In this section we will discuss the devices and methods used for the generation of standard AM wave based on the nonlinear properties.
- The circuit that generates the AM waves is called as amplitude modulator and we will discuss two modulator circuits namely,

  1. Square law modulator 2. Switching modulator

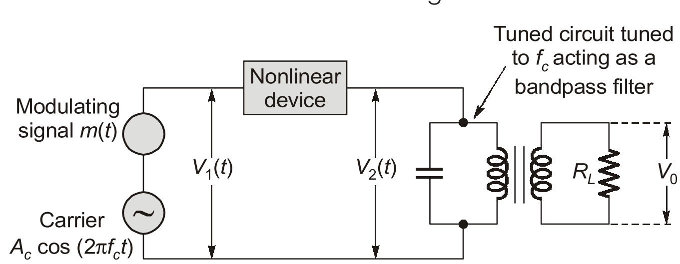
2.3.1 Square-Law Modulator

A square-law modulator requires three features:

- A means of summing the carrier and modulating waves
- A nonlinear element and
- A band-pass filter

For extracting the desired modulation products. Semiconductor diodes and transistors are the most common nonlinear devices used for implementing square-law modulators. The filtering requirement is usually satisfied by using a single or double tuned filter.

- The square law modulator circuit is as shown in figure below. It consists of the following.



When a non-linear element such as diode is suitably biased and operated in a restricted portion of its characteristic curve, we can represent the output by a square law "In the figure".

$$V_2(t) = a_1 V_1(t) + a_1 V_1^2(t)$$

Where a_1 and a_2 are constants. The input voltage $V_1(t)$ consists of the carrier wave plus the modulating wave, that is,

$$V_1(t) = A_c\cos 2\pi f_c t + m(t)$$

Therefore,
$$v_2(t) = av_1(t) + b\,v_1^2(t)$$
$$v_2(t) = a[m(t) + A_c\cos(2\pi f_c t)] + b[m(t) + A_c\cos(2\pi f_c t)]^2$$
$$v_2(t) = am(t) + aA_c\cos(2\pi f_c t) + b[m^2(t) + 2m(t)A_c\cos(2\pi f_c t) + A_c^2\cos^2(2\pi f_c t)]$$
$$= \underset{(1)}{am(t)} + \underset{(2)}{aA_c\cos(2\pi f_c t)} + \underset{(3)}{bm^2(t)} + \underset{(4)}{2bm(t)A_c\cos(2\pi f_c t)} + \underset{(5)}{bA_c^2\cos^2(2\pi f_c t)]}$$

Student's Assignment

Q.1 Consider the amplitude modulated (AM) signal $A_c \cos \omega_c t + 2 \cos \omega_m t \cos\omega_c t$. For demodulating the signal using envelope detector, the minimum value of A_c should be

(a) 2 (b) 1
(c) 0.5 (d) 0

Q.2 The Hilbert transform of $\cos \omega_1 t + \sin \omega_2 t$ is

(a) $\sin\omega_1 t - \cos\omega_2 t$ (b) $\sin\omega_1 t + \cos\omega_2 t$
(c) $\cos\omega_1 t - \sin\omega_2 t$ (d) $\sin\omega_1 t + \sin\omega_2 t$

Q.3 A given AM broadcast station transmits an average carrier power output of 40 kW and uses a modulation index of 0.707 for sine wave modulation. What is the maximum (peak) amplitude of the output if the antenna is represented by a 50 Ω resistive load?

(a) 50 kV (b) 50 V
(c) 3.414 kV (d) 28.28 kV

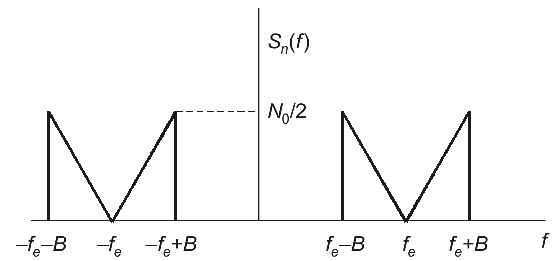
Q.4 In a trapezoidal display of modulation, the ratio of short side to long side is 0.65. The modulation percentage is

(a) 65 (b) 35
(c) 21 (d) 50

Q.5 c(t) and m(t) are used to generate an AM signal. The modulation index of the generated AM signal is 0.5. Then the quantity $\dfrac{\text{Total sideband power}}{\text{Carrier power}}$ is

(a) 1/2 (b) 1/4
(c) 1/3 (d) 1/8

Q.6 Consider the following Amplitude Modulated (AM) signal, where $f_m < B$,
$x_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t$.
The AM signal gets added to a noise with Power Spectral Density $S_n(f)$ given in the figure below. The ratio of average sideband power to mean noise power would be:



(a) $\dfrac{25}{8 N_0 B}$ (b) $\dfrac{25}{4 N_0 B}$
(c) $\dfrac{25}{2 N_0 B}$ (d) $\dfrac{25}{N_0 B}$

Q.7. The amplitude modulated wave form $s(t) = A_C[1 + K_a m(t)] \cos \omega_C t$ is fed to an ideal envelope detector. The maximum magnitude of $K_a m(t)$ is greater than 1. Which of the following could be the detector output?

(a) $A_c m(t)$ (b) $A_c^2\left[1+K_a m(t)\right]^2$
(c) $\left|A_c\left[1+K_a m(t)\right]\right|$ (d) $A_c\left[1+K_a m(t)\right]^2$

Q.8 For an AM wave, the maximum voltage was found to be 10 V and the minimum voltage was found to be 5 V. The modulation index of the wave would be

(a) 0.33 (b) 0.52
(c) 0.40 (d) 0.1

Q.9 If the radiated power of AM transmitter is 10 kW, the power in the carrier for modulation index of 0.6 is nearly

(a) 8.24 kW (b) 8.47 kW
(c) 9.26 kW (d) 9.6 kW

Q.10 The modulation index of an AM wave is changed from 0 to 1. The transmitted power is

(a) Unchanged
(b) Halved
(c) Increased by 50%
(d) Quadrupled

Q.11 In an SSB transmitter one is most likely to find
(a) Class-*C* audio amplifier
(b) Tuned modulator
(c) Class-*B RF* amplifier
(d) Class-*AB* power amplifier

Q.12 A composite signal $x_c(t)$ is expressed as:
$x_c(t) = A_c \cos \omega_c t - A_m \cos(\omega_c - \omega_m)t + A_m \cos(\omega_c + \omega_m)t$
Which of the following methods can be employed to retrieve the sinusoidal component at ω_m from $x_c(t)$?
(a) An envelop detector, square law detector
(b) Only a discriminator
(c) Only a square law detector
(d) Only an envelop detector

Q.13 An amplitude modulated signal occupies a frequency range from 395 kHz to 405 kHz. It can be demodulated by which of the following ?
(a) Using an envelope detector and filter
(b) Multiplying with a 395 kHz local signal
(c) Multiplying with a 405 kHz local signal
(d) Low pass filtering with cut off at 400 kHz

Q.14 Amplitude modulation is used for broadcasting because
(a) it is more noise immune than other modulation systems.
(b) compared with other system it require less transmitting power.
(c) its use avoids receiver complexity
(d) no other modulation system can provide the necessary BW for high fidelity.

Q.15 The maximum power efficiency of an AM modulator is
(a) 25% (b) 50%
(c) 33% (d) 100%

Q.16 Which of the following demodulator(s) can be used for demodulating the signal
$x(t) = 5(1 + 2\cos 200\pi t)\cos 20000\pi t$
(a) Envelope demodulator
(b) Square-law demodulator
(c) Synchronous demodulator
(d) None of the above

Q.17 The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is
(a) the in-phase component
(b) the quadrature-component
(c) zero
(d) the envelope

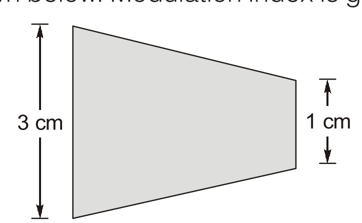
Q.18 Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel band-width?
(a) VSB (b) DSB-SC
(c) SSB (d) AM

Q.19 A message signal $m(t) = A_m \sin(2\pi f_m t)$ is used to modulate the phase of a carrier $A_c \cos(2\pi f_c t)$ to get the modulated signal $y(t) = A_c \cos(2\pi f_c t + m(t))$. The bandwidth of $y(t)$
(a) depends on A_m but not on f_m
(b) depends on f_m but not on A_m
(c) depends on both A_m and f_m
(d) does not depends on A_m or f_m

Q.20 The most commonly used filters in SSB generation are
(a) mechanical (b) RC
(c) LC (d) low-pass

Q.21 One of the following methods cannot be used to remove the unwanted sideband in SSB. This is the
(a) filter system (b) phase-shift method
(c) third method (d) balanced modulator

Q.22 Consider the Trapezoidal pattern for AM wave shown below. Modulation index is given by



(a) 33% (b) 50%
(c) 75% (d) 100%

ANSWER KEY // **STUDENT'S ASSIGNMENT**

1. (a)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (d)
6. (b)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (c)
11. (c)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (b)
16. (c)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (a)
21. (d)	**22.** (b)			

HINTS & SOLUTIONS / STUDENT'S ASSIGNMENT

1. (a)

Modulated signal,

$$\phi_{AM}(t) = A_c \cos \omega_c t + 2\cos \omega_m t \cos \omega_c t$$
$$= [A_c + 2 \cos \omega_m t] \cos \omega_c t$$

Condition for envelope detection of an AM signal is,

$$= A_c + 2 \cos \omega_m t \geq 0$$

For $\cos \omega_m t = -1$ (minimum value of $\cos\omega_m t$),

$$A_c - 2 \geq 0$$

$$\boxed{A_c \geq 2}$$

2. (a)

$$\cos \omega_1 t \xleftrightarrow{\text{H.T.}} \sin \omega_1 t$$

$$\sin \omega_2 t \xleftrightarrow{\text{H.T.}} -\cos \omega_2 t$$

3. (c)

Given: $P_c = 40$ kW, $m_a = 0.707$ and $R_L = 50\ \Omega$

We know, $m_a = \dfrac{A_m}{A_c}$

Also, $P_c = \left(\dfrac{A_c}{\sqrt{2}}\right)^2 \times \dfrac{1}{R_L}$

$\Rightarrow$ $A_c = 2000$ V

Now, $A_m = m_a A_c$

$= 0.707 \times 2000 = 1414$ V

But, we have to find peak value of AM wave

i.e. $A_m + A_c = 2000 + 1414$

$= 3414 \text{ V} = 3.414 \text{ kV}$

4. (c)

Given that,

$$\frac{L_2}{L_1} = 0.65$$

$\Rightarrow$ $L_2 = 0.65\, L_1$

$\therefore$ $m_a = \dfrac{L_1 - L_2}{L_1 + L_2} = \dfrac{0.35}{1.65}$

$= 0.212 = 21.2\%$

5. (d)

Given, $m_a = 0.5$

Find, $\dfrac{\text{Total sideband power}}{\text{Carrier power}} = \dfrac{P_c \dfrac{m_a^2}{2}}{P_c}$

$$= \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} = \frac{1}{8}$$

6. (b)

Given that,

$A_c = 10$

$m_a = 0.5$

$\therefore$ Average sideband power

$$= P_{SB} = P_c \cdot \frac{m^2}{2} = \frac{100}{2} \times \frac{0.5^2}{2} = \frac{25}{4}$$

Now, from the PSD of noise,

P_n = mean noise power

$$= \left[\frac{1}{2} \times B \times \frac{N_0}{2}\right] \times 4 = BN_0$$

$$\therefore \quad \frac{P_{SB}}{P_n} = \frac{25}{4N_0B}$$

7. (b)

When, $k_a m(t) > 1$

Non-linear distortion would occurs i.e. infinite harmonics are present.

So, detector output of $s(t) \Rightarrow s^2(t), s^3(t)$.....

$\therefore$ Output $= A_c^2[1 + K_a m(t)]^2$

8. (a)

Since, $m_a = \dfrac{V_{max} - V_{min}}{V_{max} + V_{min}} = \dfrac{10-5}{10+5} = 0.33$

9. (b)

From power relation equation,

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$\Rightarrow$ $P_c = \dfrac{10 \text{ kW}}{1 + \dfrac{(0.6)^2}{2}} = 8.47$ kW